# Fighting Pirates 2.0 

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## Introduction

- In EUROCRYPT 2009, Billet and Phan presented Traitors collaborating in public: Pirates 2.0.
- This was a new attack model agains $\dagger$ tracing and revoking schemes.
- In this work we present measures to deal with some of these attacks.


## 1. Background

- Broadcast encryption -CS and SD
-Traitor tracing


## The Broadcast Encryption Problem

- A center BC broadcast a msg to a set $U$ of $N$ receivers
- A subset $R$ of them are revoked and should
 not be able to decrypt the msg
- $R$ changes from time to time
- We will focus on stateless receivers


BC
msg

- revoked
o non-revoked


## Subset Cover Framework [NNLO1]

- Framework encapsulates many previous schemes
- Underlying collection of subsets (of users/devices)

$$
S_{1}, S_{2}, \ldots, S_{W} \quad S_{j} \subseteq U
$$

- Each subset $S_{j}$ is associated with a long-lived key $L_{j}$ - A user $u \in S_{j}$ should be able to deduce $L_{j}$ from its secret information $s k_{u}$


## The Broadcast Algorithm

- Choose a session key K
- Given $R$, find a partition of $U \backslash$ Rinto disjoint sets

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{i}_{1}}, \mathrm{~S}_{\mathrm{i}_{2}}, \ldots, \mathrm{~S}_{\mathrm{i}_{\mathrm{m}}} \\
& U \backslash R=\cup \mathrm{S}_{\mathrm{i}_{\mathrm{j}}}
\end{aligned}
$$

with associated keys $\mathrm{L}_{\mathrm{i}_{1}}, \mathrm{~L}_{\mathrm{i}_{2}}, \ldots, \mathrm{~L}_{\mathrm{i}_{\mathrm{m}}}$

- Encrypt message $M$

$$
\begin{array}{l|l}
{\left[\mathrm{i}_{1}, \mathrm{i}_{2}, \ldots, \mathrm{i}_{\mathrm{m}}\right], \quad \mathrm{C}_{1}=\mathrm{E}_{\mathrm{Li}_{1}}(\mathrm{~K}), \ldots, \mathrm{C}_{\mathrm{m}}=\mathrm{ELi}_{\mathrm{L}}(\mathrm{~K})} & \mathrm{F}_{\mathrm{K}}(\mathrm{M}) \\
\hline
\end{array}
$$

## Complete Subtree (CS)



$$
\operatorname{sk}_{\mathrm{u}}=\left\{\left(0, \mathrm{~L}_{0}\right),\left(1, \mathrm{~L}_{1}\right),\left(4, \mathrm{~L}_{4}\right),\left(10, \mathrm{~L}_{10}\right),\left(21, \mathrm{~L}_{21}\right),\left(44, \mathrm{~L}_{44}\right)\right\}
$$

## Subset Difference (SD)


$S_{i, j}$
$S_{i, j}=$ Set of all leaves in the subtree of $V_{i}$ but not in $V_{j}$

## Key-assignment for SD

- Naive key-assignment: each user must store too many keys, one for each $\mathrm{S}_{\mathrm{ij}}$
- To improve this, a pseudorandom generator is used for key derivation : each user stores only $O\left((\log N)^{2}\right)$ labels
- From labels and PRG, user covered by $S_{i j}$ can derive key $L_{i j}$


## Traitor tracing

- traitors: users that collude to produce a pirate decoder
- tracing procedure : from a pirate decoder the identity of at least one traitor is revealed
- CS and SD feature a tracing procedure:
- a traitor is identified or
- a new cover is computed (safe for the pirate decoder)

2. Pirates 2.0 attack

## Pirates 2.0: basic features

- Public collusion.
- Partial contribution.
- Anonymity guarantee.
- Large coalitions.
- Imperfect decoders.


## Pirates 2.0: the model

- Contribution C: publicly available set which collects the info traitors give
- Extraction function: function of the sk of a traitor which is added to $C$
- Anonymity level of a traitor T: \# of users which could have contributed to $C$ precisely the same info as $T$


## Pirates 2.0: the schemes

Schemes attacked in [BPO9]:

- subset cover framework
- analysis for CS and SD
- code based schemes

Our work: countermeasures for CS and SD

## Pirates 2.0 attack on CS

- Extraction functions are projections $s k_{T}=\left\{\left(i, L_{i}\right)\right\}_{i} \Rightarrow f_{i}(s k)=L_{i}$
- Traitors contribute with keys corresp. to the upper levels of the tree.
- These subtrees cover a large \# of users $\Rightarrow$ high anonymity level


## Contributed info (1 traitor)


contribution $=\left\{\mathrm{L}_{0}, \mathrm{~L}_{1}, \mathrm{~L}_{4}\right\}$

## Contributed info (>1 traitor)

## users



- traitors
contribution $=\left\{\mathrm{L}_{0}, \ldots, \mathrm{~L}_{6}\right\}$


## Pirates 2.0 attack on CS

Theorem [BPO9] :

- system with $N$ users
- $r$ revoked users
- $d \log d$ randomly selected traitors
- length of ciphertext header < $d(N-r) / N$

Then:

- successful pirate decoder (high prob.)
- anonymity level for traitors: $N / d$ Analog result for SD


## 3. Partial measures

## Partial measure for CS :

 hiding labels- Attack is successful because users know the level of their keys.
- Idea: hide the level
- $B C$ sends to user u covered by subtree $S_{i}$ $\left(\pi(i), L_{i}\right)$ instead of ( $\mathrm{i}, \mathrm{L}_{\mathrm{i}}$ )
where $\pi$ is a secret permutation of labels
- Broadcast ( $\pi(\mathrm{i}), \mathrm{E}_{\mathrm{Li}}(\mathrm{K})$ )


# Partial measure for CS: hiding labels 

## Cons:

- By public collaboration, traitors can estimate the level of their keys.
Pros:
- A traitor must trust the others.
- Traitors lose the anonimity guarantee.
- "Cheap" to implement.


## Partial measure for CS : or-based construction

- Idea: use the OR-protocols from [GSY99] to reduce anonimity level
- For each subtree $S_{i}, B C$ fixes set of keys

$$
K_{i}=\left\{L_{i 1}, \ldots, L_{i m}\right\}
$$

and a prob. dist. $D_{i}$ over $K_{i}$

- User u covered by $S_{i}$ receives a single key $L_{i j}$ according to $D_{i}$
- All keys in $K_{i}$ are used to broadcast


## Partial measure for CS : or-based construction

Cons:

- Total \# of gen. keys grows by m factor
- Ciphertext length grows by $m$ factor

Pros:

- \# keys per user remains the same
- anon. level is reduced
- anon. guarantee is lost (only probabilistic)


## 4. Hybrid CS and SD

## Hybrid CS scheme: Idea

Combine two constructions:

- CS scheme from [NNL01].
- Polynomial-based scheme from [NPOO].


## Hybrid CS: Parameters

- $G=\langle g\rangle$ : group of order $q$ with hard DDH.
- threshold value $\dagger>0$
- (public) reconstruction values

$$
\left\{I_{1}, \ldots, I_{+}\right\} \text {in } Z_{q} \backslash\{0\}
$$

- User u gets $I_{u}$ in $Z_{q} \backslash\left\{0, I_{1}, \ldots, I_{+}\right\}$


## Hybrid CS: Setup

For each subtree $S_{i}, B C$

- chooses (secret) t-degree polymial $P_{i}(x) \leftarrow_{\$} Z_{q}[x]$
- sends to each user u covered by $S_{i}$ (i, $P_{i}\left(I_{u}\right)$ )


## Hybrid CS: Broadcast

For new session, $B C$

- chooses session key K
- computes a cover $S=\left\{S_{i}\right\}$ for leg. users
- for each subtree $S_{i}$ in $S$ :

1. $r_{i} \leftarrow_{\$} Z_{q}$
2. $\left.\forall j=1, \ldots, t \quad d_{i j}:=g^{r i p(I T}\right)$
3. $K_{i}=g^{r i p i(0)}$
4. broadcasts ( $\mathrm{i}, \mathrm{g}^{\mathrm{ri}},\left\{\mathrm{d}_{\mathrm{i}}\right\}_{j}, \mathrm{E}_{\mathrm{k} i}(\mathrm{~K})$ )

- broadcasts $F_{K}(M)$


## Hybrid CS: Decryption

Leg. user $u$, from
broadcast: $\left(\mathrm{i}, \mathrm{g}^{\mathrm{ri}},\left\{\mathrm{d}_{\mathrm{ij}}:=g^{\mathrm{ri} \mathrm{Pi}\left(\mathrm{I}_{\mathrm{j}}\right)}\right\}, \mathrm{E}_{\mathrm{ki}}(\mathrm{K})\right)$ u info: ( $\left.i, P_{i}\left(I_{u}\right)\right), I_{u}$
(public) values: $\left\{I_{1}, \ldots, I_{+}\right\}$
computes the subtree key $\mathrm{K}_{i}:=g^{\text {ri Pi }}(0)$ by
"polynomial interpolation in the exponent". Then recovers session key $K$

## Hybrid SD scheme: Idea

Also combine the 2 constructions:

- SD scheme from [NNL01].
- Polynomial-based scheme from [NPOO].

Not an immediate generalization of previous construction:

- We preserve the pseudorandom key generation which allows each user to store only $O\left((\log N)^{2}\right)$ labels.


## Hybrid SD: Parameters

- $G=\langle g\rangle$ : group of order $q$ with hard DDH.
- threshold value $\dagger>0$
- (public) reconstruction values

$$
\left\{I_{1}, \ldots, I_{+}\right\} \text {in } Z_{q} \backslash\{0\}
$$

- User u gets $I_{u}$ in $Z_{q} \backslash\left\{0, I_{1}, \ldots, I_{+}\right\}$


## Hybrid SD: Setup

$B C$ generates an instance of SD with $Z_{q}$ as set for keys $L_{i j}$
Then, for each subtree $S_{i}, B C$

- chooses (secret) t-degree polymial

$$
P_{i}(x) \leftarrow_{\$} Z_{q}[x]
$$

- sends to each user u covered by $S_{i, *}$
(i, $P_{i}\left(I_{u}\right)$ ) and
labels that SD assigns to him


## Hybrid SD: Broadcast

For new session, $B C$

- chooses session key K
- computes a cover $S=\left\{S_{i j}\right\}$ for leg. users
- for each subtree $S_{i j}$ in $S$ :

1. $r_{i} \leftarrow_{\$} Z_{q}$
2. $\forall k=1, \ldots, t \quad d_{\mathrm{ijk}}:=g^{r i} \mathrm{P}(\mathrm{I} \mathrm{K}) \mathrm{Lij}$
3. $K_{i j}:=g^{r i} P_{i}(0) L i j$
4. broadcasts (ij, gri, $\left.\left\{\mathrm{d}_{\mathrm{ijk}}\right\}_{k,}, E_{\mathrm{kij}}(\mathrm{K})\right)$

- broadcasts $F_{K}(M)$


## Hybrid SD: Decryption

- Again, leg. user u recovers subtree key $\mathrm{K}_{\mathrm{ij}}$ by "polynomial interpolation in the exponent".
- Then u recovers session key $K$


## Hybrid CS and SD: Analysis

- Each pair $\left(i, P_{i}\left(I_{u}\right)\right)$ determines univocally user u
- Therefore the Pirates 2.0 strategy that uses projection functions does not work anymore, as anonymity level drops to 1 (traitor can be traced)


## Hybrid CS and SD: Analysis

- We also prove that our schemes satisfy the key-ind property in the Subset-Cover framework.
- This implies that they are secure against arbitrary coalitions of revoked users.
- They are also as efficient as CS and SD, in terms of key storage and bandwidth (with a t factor growth)


## Hybrid CS and SD: Analysis

Price to pay:

- Broadcast and decryption computations are more expensive than ones in CS and SD (exponentiations)
- $\dagger+1$ users covered by subtree $S_{i}$ can compute and distribute $P_{i}(0)$, which allows to decrypt if $S_{i}$ is used


## Hybrid CS and SD: Analysis

Advantages:

- Pirates 2.0 with proj. func. are traced
- Secure against arb. coa. of rev. users
- Efficient as CS and SD both in:
- Key storage
- Bandwith (asymptotically)


## Open problems

- It is of interest to formally define a security model which covers all possible Pirates 2.0 attacks
- and find and prove schemes (existing or new) to be secure in this extended model.


## Thank you!

 Questions?